The extended Unruh effect in the Kruskal spacetime

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Originally, the Unruh effect is considered in the Minkowski spacetime. That is, for a uniformly accelerated observer in the Minkowski spacetime, there will be an event horizon, and the observer will detect radiation from it. In this paper, we extend the Unruh effect to the Kruskal spacetime. After defining some special null hypersurface in the Kruskal spacetime, we find that it can be considered as the event horizon for some accelerated observers, too. Moreover, there is radiation from it. In addition, our result shows that these observers by definition are 3-accelerated observers in the Schwarzschild space. Thus, the conculsion may be available in the astronomical exploration.

Key words: Unruh effect, event horizon, radiation

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I. INTRODUCTION

In 1976, for the sake of gaining more insight into the nature of the Hawking effect, Unruh considered an observer that uniformly accelerated in the Minkowski space-time[1]. And yet, another interesting result was also obtained, that is, the observer will be excited as if vacuum were hot, with a temperature proportional to the acceleration. This is the well-known "Unruh effect".

Since this effect was discovered, it has caused many revolutions on our concepts of physics. And probably the most significant one is the notion of the vacuum. Different from our empirical concept, the vacuum is dependent on the observer[2, 3]. On the other hand, the Unruh effect origins from the research on Hawking effect[4, 5], and it can be even equivalent to the temperature effect measured by a stationary detector close to the event horizon of a black hole if the mass of the black hole tends to infinity[1]. But the Unruh effect can also be considered to be distinct from the Hawking effect of spontaneous particle creation by black holes both physically and mathematically[6]. Thus, giving a further extension on the Unruh effect is of sense. According to the kind of the space-times, there have already been a lot of extensions both in the flat spacetimes and the curved space-times[7, 8, 9, 10, 11, 12, 13]. In this paper, we reinvestigate the Unruh effect in the curved Kruskal space-time. Not as before, our start is not the particular observer but some special null hypersurface. That is, we first define some special null hypersurface in the Kruskal spacetime, and then define the corresponding observers whose event horizon can be just the null hypersurface defined.

The rest of the paper is organized as follows. In section 2, we first define the null hypersurface and its corresponding observers, and then calculate the proper accelerations of the observers. In section 3, using the Damour-Ruffini method[14], we prove that there is radiation from the null hypersurface which has been defined in section 2, and we also obtain the thermal spectrum. Finally, in section 4, we give a brief conclusion and discussion on our result.

II. THE NULL HYPERSURFACE AND ITS CORRESPONDING OBSERVERS BY DEFINITION

Usually, the line element of the Kruskal spacetime is[15]

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}}(-dT^2 + dX^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{1}$$

where

$$(\frac{r}{2M} - 1)e^{\frac{r}{2M}} = X^2 - T^2. \tag{2}$$

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In our paper, we set our discussion only in the region of the Schwarschild spacetime. Thus, we first define the null hypersurface in the Kruskal coordinates that

$$T = \pm(X - X_0). \tag{3}$$

where X_0 is a positive constant. From this definition, It's easily seen that the event horizon of the Schwarschild spacetime can be obtained if we set $X_0 = 0$.

According to (3), the corresponding observer can be defined

$$(X - X_0)^2 - T^2 = e^{2a\xi_0}, \theta = \theta_0, \varphi = \varphi_0. \tag{4}$$

where $a, \xi_0, \theta_0, \varphi_0$ are also constants. In fact, if we let the ξ_0 be different constant, we can define a family of observers. One of the properties of these observers is that their world lines are all hyperbolic spacetime curves. Furthermore, the null hypersurface defined in (3) is just the event horizon of them, which can be easily seen from the spacetime diagram (Figure 1). On the other hand, we can define a new coordinate system $\{\eta, \xi, \theta, \varphi\}$ through these observers that

$$X = X_0 + e^{a\xi} cha\eta, T = e^{a\xi} sha\eta. \tag{5}$$

Thus, the new metric in this coordinates is

$$\begin{aligned} ds^2 &= \frac{32M^3}{r} e^{-\frac{r}{2M}} e^{2a\xi} a^2(-d\eta^2 + d\xi^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \\ &= g_{00}d\eta^2 + g_{11}d\xi^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \end{aligned} \tag{6}$$

where

$$(\frac{r}{2M} - 1)e^{\frac{r}{2M}} = X_0^2 + 2e^{a\xi} X_0 ch(a\eta) + e^{2a\xi}. \tag{7}$$

From which, we find that the family of the observers are resting in the new frame (6), and can obtain the conclusion that the null hypersurface defined in (3) is the event horizon of those observers defined by (4) again. Because the event horizon is $\xi = -\infty$, and it is just the null hypersurface defined in (3).

In the following, for the aim of getting more details about these observers, we will calculate their 4-velocity and proper acceleration. The tangent vector of the observer's worldline ($\xi = \xi_0$) in (5) can be written as

$$(\frac{\partial}{\partial\eta})^a = \frac{\partial x^\mu}{\partial\eta}(\frac{\partial}{\partial x^\mu})^a = a[T(\frac{\partial}{\partial X})^a + (X - X_0)(\frac{\partial}{\partial T})^a]. \tag{8}$$

And if reparameterize the world line with the proper time τ of the observer, we obtain

$$(\frac{\partial}{\partial\eta})^a = \frac{d\tau}{d\eta}(\frac{\partial}{\partial\tau})^a = \gamma(\frac{\partial}{\partial\tau})^a, \gamma \equiv \frac{d\tau}{d\eta}. \tag{9}$$

where $(\frac{\partial}{\partial\tau})^a$ is the 4-velocity. Using the unitary property of the 4-velocity and the metric in (1), we can obtain

$$\begin{aligned} \gamma^2 &\equiv (\frac{d\tau}{d\eta})^2 = a^2 e^{2a\xi_0} \frac{32M^3}{r} e^{-\frac{r}{2M}}, \\ \Gamma^0_{00} &= \Gamma^0_{11} = \Gamma^1_{01} = \frac{2(r+2M)MT}{r^2 e^{\frac{r}{2M}}}, \\ \Gamma^0_{01} &= \Gamma^1_{00} = \Gamma^1_{11} = -\frac{2(r+2M)MX}{r^2 e^{\frac{r}{2M}}}. \end{aligned} \tag{10}$$

Thus, the 4-velocity and the 4-acceleration are

$$
\begin{aligned}
T^a &= (\frac{\partial}{\partial \tau})^a = \frac{1}{\gamma}(\frac{\partial}{\partial \eta})^a = \frac{a}{\gamma}[T(\frac{\partial}{\partial X})^a + (X - X_0)(\frac{\partial}{\partial T})^a] \\
A^b &= \frac{e^{-2a\xi}}{\frac{32M^3}{r}e^{-\frac{r}{2M}}} B[T(\frac{\partial}{\partial T})^b + (X - X_0)(\frac{\partial}{\partial X})^b].
\end{aligned} \tag{11}
$$

where

$$
B \equiv 1 - [(X - X_0)X_0 + e^{2a\xi_0}]\frac{2M(r + 2M)}{r^2}e^{-\frac{r}{2M}}. \tag{12}
$$

And the proper acceleration of the observer is

$$
A \equiv |A^a| = \frac{1}{e^{a\xi_0}}(\frac{32M^3}{r}e^{-\frac{r}{2M}})^{-1/2}B. \tag{13}
$$

From (8) and (11), we can obtain that the definition in (4) can be truly considered as one observer's worldline. Because the worldline is timelike, which can be easily seen from the expression of the tangent vector in (8) (can also be seen directly from the spacetime diagram). And the corresponding observer by definition is an accelerated observer in the Kruskal spacetime whose proper acceleration is changed along with its proper time's evolution, which is different from that of the static observer in the Schwarschild spacetime, a constant proper acceleration. In addition, the proper acceleration of the event horizon can be considered as positive infinite in the limit $\xi_0 \to -\infty$ in (13), which is a general property of the event horizon.

III. RADIATION FROM THE EVENT HORIZON

In the above section, we have showed that the null hypersurface is just the event horizon for the corresponding observers. But is there radiation from the event horizon? And if there is radiation, what's the spectrum? In the following, we will discuss these two questions in details.

As we know, there are many approaches to prove the radiation from the event horizon[5, 12, 14, 16]. In this paper, we use the Damour-Ruffini method[14]. In the curved space-times, the Klein-Gordon equation is

$$
\frac{1}{\sqrt{-g}}\frac{\partial}{\partial X^\mu}(\sqrt{-g}g^{\mu\nu}\frac{\partial \Psi}{\partial X^\nu}) - \mu^2\Psi = 0. \tag{14}
$$

substituting (6) into (14), we obtain

$$
[-\frac{\partial}{\partial \eta}(r^2\frac{\partial \Psi}{\partial \eta}) + \frac{\partial}{\partial \xi}(r^2\frac{\partial \Psi}{\partial \xi})] \cdot \frac{1}{g_{11}} + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial \Psi}{\partial \theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\Psi}{\partial \varphi^2} = \mu^2 r^2\Psi. \tag{15}
$$

If we let the $\Psi(\eta, \xi, \theta, \varphi)$ be written into the following form

$$
\Psi(\eta, \xi, \theta, \varphi) = R(\eta, \xi)Y_{lm}(\theta, \varphi). \tag{16}
$$

where $Y_{lm}(\theta, \varphi)$ is the usual spherical harmonics. Then we can simplify the equation (15) that

$$
-\frac{\partial}{\partial \eta}(r^2\frac{\partial R}{\partial \eta}) + \frac{\partial}{\partial \xi}(r^2\frac{\partial R}{\partial \xi}) = [\mu^2 r^2 R - l(l+1)R]g_{11}. \tag{17}
$$

On the other hand, from (7), we obtain

$$
\frac{\partial r}{\partial \xi} = \frac{2ae^{a\xi}X_0 ch(a\eta) + 2ae^{2a\xi}}{\frac{r}{(2M)^2}e^{\frac{r}{2M}}}, \frac{\partial r}{\partial \eta} = \frac{2ae^{a\xi}X_0 ch(a\eta)}{\frac{r}{(2M)^2}e^{\frac{r}{2M}}}. \tag{18}
$$

Thus, when $\xi = -\infty$, the location of the event horizon, (17) will be

$$-\frac{\partial^2 R}{\partial \eta^2} + \frac{\partial^2 R}{\partial \xi^2} + 2r[\frac{\partial R}{\partial \xi} - \frac{\partial R}{\partial \eta}]\frac{\partial r}{\partial \eta} = 0. \tag{19}$$

and the solution is

$$R_\omega^{in} = e^{-i\omega(\eta+\xi)}, R_\omega^{out} = \frac{C}{r}e^{-i\omega(\eta-\xi)} \quad \text{(here C is the normalized factor).} \tag{20}$$

In order to find out more properties of the solutions of the particles going out (out-wave) and into the event horizon (in-wave) in (20), we use the advanced Eddington-Finkelstein coordinates which is defined by $\nu = \eta + \xi$. Thus, the (20) becomes

$$R_\omega^{in} = e^{-i\omega\nu}, R_\omega^{out} = \frac{C}{r}e^{2i\omega\xi}e^{-i\omega\nu}. \tag{21}$$

From it, It's easily to find that the out-wave is not analytic in the horizon, while the in-wave is analytic. For getting an analytic solution of the out-wave, we can use the method of the analytical extension through the complex plane. Before doing this, we first give another coordinates transformation which is

$$\xi = \frac{1}{a}\ln\rho. \tag{22}$$

This transformation's most advantage is that it will change the location of event horizon from $\xi = -\infty$ into $\rho = 0$, and it would be more convenient for us to find out the analytic solution of R_ω^{out}.

After giving the coordinates transformation, the R_ω^{out} is

$$R_\omega^{out} = \frac{C}{r}\rho^{2i\omega/a}e^{-i\omega\nu} \quad (r > r_H). \tag{23}$$

and the metric in (6) becomes

$$ds^2 = \frac{32M^3}{r}e^{-\frac{r}{2M}}\rho^2 a^2(-d\eta^2 + d\xi^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{24}$$

From which, it's easily obtained that if we take this exchange $\rho \to -\rho e^{-i\pi/2}$ through the complex plane, the analytic solution of $\tilde{R}_\omega^{out}$ inside the horizon can be found that

$$\tilde{R}_\omega^{out} = \frac{C}{r}e^{\pi\omega/a}e^{2i\omega\xi}e^{-i\omega\nu} \quad (r < r_H). \tag{25}$$

Thus, if we take the Heaviside function Y into the use, the solution of the out-wave $\hat{R}_\omega^{out}$ which is analytic both inside and outside the event horizon can be described

$$\hat{R}_\omega^{out} = N_\omega[Y(r - r_H)R_\omega^{out} + Y(r_H - r)\tilde{R}_\omega^{out}]. \tag{26}$$

where N_ω is a normalization factor. Using the relation for the Bose particles

$$< \hat{R}_{\omega_1}^{out}, \hat{R}_{\omega_2}^{out} >= -\delta(\omega_1 - \omega_2) \tag{27}$$

we obtain

$$N_\omega^2 = (e^{2\pi\omega/a} - 1)^{-1} \tag{28}$$

Therefore, the temperature is

$$T = \frac{a}{2\pi} \tag{29}$$

which implicates that it truly exists the radiation from the event horizon for those observers, and the spectrum is also the Planck thermal spectrum.

IV. CONCLUSION AND DISCUSSION

As we know, there are many similarities between the Unruh effect and the Hawking radiation. And probably the most remarkable similarity is that the worldlines of the observers (the static observers in the Schwarschild spacetime, while the uniform linear acceleration observers in the Minkowski spacetime) are all hyperbolic curves in the usual coordinates which can be over the maximal extension spacetime. However, it isn't the only one type of hyperbolic curves in the maximal extension spacetime. Thus, defining a new type of hyperbolic curves and researching the properties of the corresponding observers is one of our original motivations. But taken the high intrinsic symmetry of the flat spacetimes into consideration, it would be better for us to consider it in the curved spacetimes. Thus, we consider it in the Kruskal spacetime, one of the most simple curved spacetimes.

In our paper, the result shows that we can define a new type of hyperbolic curves by the definition in (4) before selecting some special null hypersurface in the curved Kruskal spacetime. Moreover, we can prove that the event horizon of the corresponding observers is just the null hypersurface by definition in (3). In fact, viewed from the spacetime's symmetry, this definition can be considered as a coordinate translation which is similar with one spatial coordinate translation in the Minkowski space-time. But different from that of the Minkowski space-time, the invariance of this coordinate translation is lost in the Kruskal spacetime. Thus, the properties of the observers by definition will be non-trivial, which is true and can be easily seen from (12) that the proper acceleration changes with the time's evolution, while it's a constant for the static observers in the Schwarschild spacetime. However, after using the Damour-Ruffini method to prove that it does exist radiation from the event horizon for these observers, we obtain a trivial result (29), which is interesting and manifests the speciality of the radiation.

In addition, another motivation of our paper is considering an interesting question about what the observer will detect if this observer accelerates in the Schwarschild space(which is relative to the static observers). And our paper may partly reply to it. As calculating in section 2, we have proved that the observers defined in (4) is a 4-accelerated observers in the Kruskal spacetime. But the proper acceleration is a constant for the static observers in Schwarschild spacetime, so the observers we defined must be a 3-accelerated observers in the Schwarschild space. In fact, if we use the Schwarschild coordinates $\{t, r, \theta, \varphi\}$ to express the worldline in (4), we will obtain

$$(\frac{r}{2M} - 1)^{1/2} e^{\frac{r}{4M}} = X_0 ch\frac{t}{4M} + \sqrt{X_0^2 sh\frac{t}{4M} + e^{2a\xi_0}}. \tag{30}$$

which shows that the observer defined in (4) is an observer which is at first close to the black hole, and then far away from it (Figure 2). Thus, our result can imply that one observer who accelerates in the Schwarschild space will also detect a radiation. However, the radiation is not from the event horizon $r = 2M$ of the Schwarschild spacetime, but from another event horizon such as the null hypersurface defined in (3), which may be available during the astronomic exploration.

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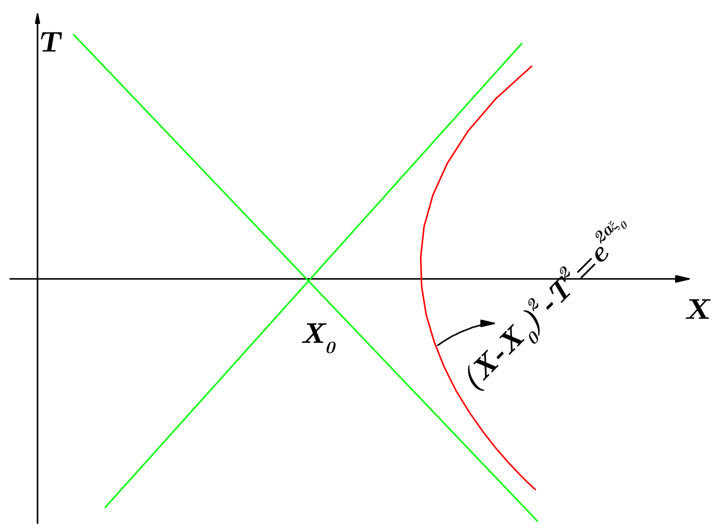

FIG. 1: (Color online) Null hypersurface and the corresponding observer by definition.

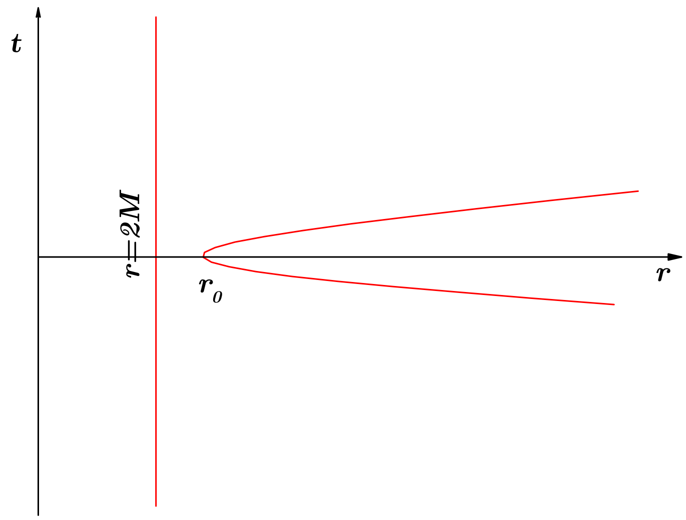

FIG. 2: (Color online) The behavior of the observer in the Schwarzschild coordinates.